COUPLING EFFECTS FOR SECONDARY SYSTEMS IN NUCLEAR POWER PLANTS

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ABSTRACT

The effect of interaction (coupling) between the Primary (P) and light secondary systems on the maximum responses of the secondary systems is studied. Some of the problems and fallacies associated with the conventional modal analysis approach are discussed and pointed out. The need to discontinue using this approach in cases where light secondary systems are in resonance with the primary system is explained.

Upper and Lower bounds for the responses of the Secondary (S) and Second-Secondary (SS) systems have been developed and presented. The upper bounds were found to be dependent on the sum of mass ratios while the lower bounds are independent of mass ratios.

Amplification Factors for a coupled P-S-SS system which accounts for mass ratios effects such as those shown in Fig (4) of the paper have been developed. These amplification factors which are developed for a particular damping value and a particular modal combination rule show a substantial reduction in secondary systems response with the increase in mass ratios.

		NOMENCLATURE
Α	=	Acceleration value of an input response spectrum (constant)
[C]	=	Damping Matrix
ĪĪI	=	Modal Damping Matrix
CSM	=	Closely Spaced Modes
[K]	=	Stiffness Matrix
[M]	=	Mass Matrix
P	=	Primary System
P ₁ , P ₂ , P ₃	=	Response of 'P' by absolute sum, SRSS, and by algebraic sum respectively
S	=	Secondary system
s ₁ , s ₂ , s ₃	=	Response of 'S' by absolute sum, SRSS, and by algebraic sum respectively
SS	=	Second-Secondary system
s_{1}, s_{2}, s_{3}	=	Response of 'SS' by absolute sum, SRSS, and by algebraic sum respectively
ω1	=	Frequency of the P-system or one of its modal frequencies
ω2	=	Frequency of the S-system or one of its modal frequencies
ω3	=	Frequency of the SS-system or one of its modal frequencies
$\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3$	=	Frequencies of the coupled P-S-SS system
μ	=	Ratio of the modal mass of the S-system to the modal mass of the P-system
Ŷ	=	Ratio of the modal mass of the SS-system to the modal mass of the S-system
ε ₁ , ε ₂ , ε ₃	=	Eigenvalues shifts (Fig 2)
^φ 1, ^φ 2, ^φ 3	=	Mode shape vectors for first, second, and third modes
β	=	Damping ratio

INTRODUCTION

Practical design considerations necessitate many systems in a Nuclear Power Plant (NPP) to be decoupled for the purpose of seismic analysis. This is customarily done for the following practical and technical reasons:

- a) The secondary system analysis is typically the responsibility of a sub-contractor while the primary system analysis is the responsibility of a main contractor. Thus the breakdown into secondary and primary systems follows the actual breakdown of responsibility.
- b) The detailed combined model of the seondary and primary systems is typically large, too expensive to run and prone to human errors.

- c) The design of the primary system typically precedes the design of the secondary systems. The dynamic data available on the secondary systems at the time of the analysis of the primary system does not justify a detailed coupled analysis.
- d) Most important for very light secondary systems compared to the primary system, the credibility of a coupled analysis may become questionable numerically. This is true whether a timehistory or a response spectrum approach is used.
- e) Major state-of-art difficulty exists in assigning modal damping values for systems which have different damping ratios. An elaborate detailed model may not be consistent with the adequacy of the damping data used in formulating the model.

Thus, the assumption of decoupling, if proven valid and resonable, will alleviate these practical and theoretical difficulties. At present there exists a number of decoupling criteria available in the industry to define to the designer the conditions under which decoupling is acceptable. These criteria (1,6,7,8,9) are based on the concepts of modal mass ratios and frequency ratios of the secondary and primary systems and are shown in Figure (1).

In many occasions the designer may encounter a light secondary system which is supported by another secondary system which in turn is supported by a primary system (e.g., a light equipment which is supported by another equipment which in turn is supported by a building). Studies for the coupling effects in this popular practical situation is almost non-existent at present. This is primarily because of the mathematical difficulties and the large number of variables encountered in the problem. To differentiate in the following discussion between the two secondary systems, the supported secondary system will be termed 'SS' (for Second-Secondary) while the supporting secondary system will be termed 'S' (for Secondary only). The letter 'P' will be used to describe the primary system. A combined system which consists of the primary and the secondary systems will be termed a P-S-SS system.

REVIEW OF RELATED WORK

Very little is found in the literature on the coupling phenomena for a P-S system in a seismic environment. Crandell and Mark (5) studied the effects of mass ratio and frequency ratios on the mean-square response to an ideal white noise input. Their results showed that the response of both the P and S systems are reduced by a coupled analysis, even if the two systems are in resonance (an exception; the relative displacement of the primary mass may increase slightly). Pickel (6) proposed some general guidelines for the maximum mass ratio for decoupling to be reasonable for each frequency ratio. Hadjian (7) suggested graphical decoupling criteria which are more consistent. Aziz and Duff (1,2) presented a family of decoupling criteria which are consistent, rationally derived, and continuous functions for any mass ratio and any frequency ratio. Newmark (12) introduced the concept of the inverse of the square root of the mass ratio as an upper bound for the ampli-

fication in the secondary system response. Gelman (10) provided interaction factors to be used in the seismic analysis based on the SRSS rule; his use of the SRSS rule is open to questioning. Suzuki (11) conducted a study for a two degree-of-freedom system with mass ratios ranging between 0 and 0.1 under twenty actual earthquakes. His results showed conclusively a substantial reduction in the response of the secondary system with mass ratio. Kelly and Sackman (3) proposed a new summation rule for the two closely spaced modes which appear in a P-S system (2). All these studies have been treating a P-S system only. Studies for a P-S-SS system have not existed to the author's knowledge. It is the objective of this paper to provide an understanding of the behaviour of this system in a seismic environment. The work is an extension of the previous work reported in References (1), (2) on a P-S system by Aziz and Duff.

EIGENVALUE ANALYSIS OF A P-S-SS SYSTEM

The starting point for any study in mass coupling effects for a P-S-SS system is the three degree-of-freedom system shown in Figure (2). Although the Figure depicts a simple three degree-of-freedom system, it should be interpreted as a modal three degree-of-freedom system representing any three modes, one for the P-system, one for the S-system and one for the SS-system. The characteristic parameters for this three degree-of-freedom system are ω_1 , the frequency of the P-system or alternatively one of its modal frequencies; ω_2 , the frequency of the S-system or alternatively one of its modal frequencies; ω_3 , the frequencies; the mass ratio μ which is defined as the ratio of the modal mass of the S-system to the modal mass of the P-system and the mass ratio γ which is defined as the ratio of the SS-system to the modal mass of the SS-system to the modal mass of the SS-system to the modal mass of the SS-system are of the one degree-of-freedom type, the modal mass ratios are the same as the actual mass ratios. This is not generally true for multi-degree-of-freedom systems.

For space limitations, only the case when $\omega_3 = \omega_2 = \omega_1$ (resonance case) will be presented here. Similar treatment can be done for the non-resonance case ($\omega_3 \neq \omega_2 \neq \omega_1$). The resonance case is more important from the practical point of view since it leads to the highest amplifications.

The coupled system shown in Figure (2) exhibits three frequencies $\overline{\omega}_1$, $\overline{\omega}_2$, and $\overline{\omega}_3$. The shift of the eigenvalues (squares of the frequencies) of the coupled system is given by ε_1 , ε_2 , and ε_3 respectively (Figure 2).

Limiting the presentation for the case of resonance and small mass ratios (i.e., $\mu < < 1$. and $\gamma < < 1$.); it can be shown that:

ε1	2 11	$-\sqrt{\mu + \gamma}$	(1)
ε2	ĩ	$\frac{\mu\gamma}{(\mu+\gamma)+\mu\gamma}$	(2)

$$\varepsilon_3 = +\sqrt{\mu + \gamma} \tag{3}$$

This suggests that the spread in the square of the coupled frequencies is dependent on both μ and γ ; (equals $2\sqrt{\mu + \gamma}$). Even when μ and γ (modal mass ratios) are as small as 0.01, the spread in the square of the coupled frequencies is 28.28% and the spread in the coupled frequencies is 14.14%. Thus a coupled model even in this case utilizing a time-history approach will produce results which are very much dependent on the specific nature of the frequency content of the timehistory used in the neighbourhood of the frequencies of the system.

MODE SHAPES AND PARTICIPATION FACTORS

The mode shapes and participation factors for the case of resonance and small values of μ and γ can be approximated by:

$$\phi_{1} = \begin{pmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{pmatrix} = \begin{pmatrix} \frac{1.0}{\sqrt{\mu + \gamma}} \\ \frac{1}{\mu} \end{pmatrix} , \quad \Gamma_{1} = \frac{\mu}{2(\mu + \gamma)}$$
(4)

$$\phi_{2} = \begin{pmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{pmatrix} = \begin{pmatrix} 1.0 \\ \frac{\mu}{\mu + \gamma} \\ -\frac{1}{\gamma} \end{pmatrix} , \Gamma_{2} = \frac{\gamma}{(\mu + \gamma)}$$
(5)

$$\phi_{3} = \begin{pmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{pmatrix} = \begin{pmatrix} \frac{1.0}{-\sqrt{\mu + \gamma}} \\ \frac{1}{\mu} \end{pmatrix} , \quad \Gamma_{3} = \frac{\mu}{2(\mu + \gamma)}$$
(6)

The following observations can be made:

(a)
$$\Gamma_1 + \Gamma_2 + \Gamma_3 = 1.0$$
 (7)

This is a direct result of the fact that $\Sigma_i \Gamma_i \phi_i$ is unity for a system subjected to ground motion

(b)
$$\lim_{\mu \to 0} \Gamma_1 = \lim_{\mu \to 0} \Gamma_3 = \frac{1}{2} \text{ (For any } \mu > 0)$$
 (8)
 $\gamma \to 0$

$$\lim_{\gamma \to 0} \Gamma_2 = 0 \qquad (For any \ \mu > 0) \qquad (9)$$

The problem becomes identical to that treated by Aziz and Duff in Ref. $\left(2\right)$

(c)
$$\lim_{\mu \to 0} \Gamma_1 = \lim_{\mu \to 0} \Gamma_3 = o$$
 (For any $\gamma > o$) (10)

$$\lim_{\gamma \to 0} \Gamma_2 = 1.0 \qquad (\text{For any } \gamma > o) \qquad (11)$$

(d)
$$\lim_{\mu \to 0} \Gamma_1 \phi_{21} = -\lim_{\mu \to 0} \Gamma_3 \phi_{23} = \infty$$
 (12)
 $\mu \to 0$ $\gamma \to 0$

$$\underset{\substack{\mu \to 0 \\ \gamma \neq 0}}{\operatorname{Lim}} \Gamma_1 \phi_{31} = \underset{\substack{\mu \to 0 \\ \gamma \neq 0}}{\operatorname{Lim}} \Gamma_3 \phi_{33} = - \underset{\substack{\mu \to 0 \\ \mu \neq 0}}{\operatorname{Lim}} \Gamma_2 \phi_{32} = \infty$$
(13)

RESPONSE ANALYSIS

To achieve an understanding for the response of the coupled P-S-SS system, an input response spectrum of constant value 'A' will be assumed (for example the constant acceleration branch of a typical ground response spectrum). The damping attached to this constant value 'A', as demonstrated later, is the average modal damping of the P, S, and SS systems. The response of the P, S, and SS systems can be obtained by different modal rules as follows:

Response of the Primary System (P): Response of 'P' by absolute sum, SRSS, and by algebraic sum will be denoted P_1 , P_2 , and P_3 respectively: (a)

$$P_{1} = (|r_{1}| + |r_{2}| + |r_{3}|)A = A$$
(14)

$$P_{2} = \sqrt{\Gamma_{1}^{2} + \Gamma_{2}^{2} + \Gamma_{3}^{2}} A < A$$
(15)

$$P_3 = (\Gamma_1 + \Gamma_2 + \Gamma_3) \quad A = A$$
 (16)

Response of the Secondary System (S): In a similar manner, S_1 , S_2 , and S_3 are defined as the response of the secondary system 'S' by absolute sum, SRSS, and by alge-(b) braic sum respectively and is given by:

$$S_1 = \frac{1}{\sqrt{\mu + \gamma}} A \tag{17}$$

$$S_2 = \frac{1}{\sqrt{\mu + \gamma}} \frac{A}{\sqrt{2}}$$
(18)

(c)

Response of the Second-Secondary System (SS): In a similar manner, S_1 , S_2 , and S_3 are defined as the response of the second-secondary system 'SS' by absolute sum, SRSS, and by algebraic sum respectively and is given by:

$$S_1 = \frac{2}{(\mu + \gamma)} A$$
 (20)

$$S_2 = \frac{\sqrt{1.5}}{(\mu + \gamma)} A$$
 (21)

$$\mathbf{s}_{3}^{\prime} = \mathbf{A} \tag{22}$$

For systems which are subjected to earthquake ground motions, the absolute sum represents an upper bound for the system response while the algebraic sum represents a lower bound. It is interesting to note that an upper bound for the S-system response is $\frac{A}{\sqrt{\mu+\gamma}}$; while an upper bound for the SS-system response is $\frac{2A}{(\mu+\gamma)}$. The lower bound response is only A for both the S and SS systems. Of

A for both the S and SS systems. Of course the upper bounds and the lower bounds are far apart. The upper bounds for the S and SS system can be improved by applying the SRSS rule which leads to a reduction of approximately 29% and 39% in these upper bounds respectively. These upper bounds are extremely useful if μ and γ are very different from each other, although both are still small quantities, since the response will be somewhat governed by the largest of them.

For small mass ratios (i.e. $\mu <<1$. and $\gamma <<1$.), the three resulting modes for the P-S-SS system, are closely spaced. Thus if an absolute sum of the modes is performed as the normal practice in the industry (13), the calculated response of the secondary systems may be unrealistically high, and actually may be higher than that obtained by a decoupled analysis. Thus a conventional modal analysis utilizing a coupled model with very small mass ratios may result in both unrealistic and erroneous results, and therefore should be discontinued. On the other hand, using a time-history analysis would lead to more realistic results. The problem is that the results are realistic for a particular time history. Even when the time history is a spectrum compatible one, the way closely spaced modes interact (add or subtract) will depend to a great extent on the nature of the time history. The fact that modes do combine differently for different time histories should not be overlooked in this regard.

COMBINATION OF TWO CLOSELY SPACED MODES

Mass coupling effects lead to closely spaced modes. It is extremely important to devise a way to combine two CSM accurately rather than by the absolute sum method which is extremely conservative for the secondary systems responses. Of particular importance is the case where the two modal contributions are equal but of opposite signs. The discussion in this section will be restricted to the case of two CSM only, which have the same amount of modal damping ' β '. The first mode contribution is Q₁, and the second mode contribution is Q₂; where Q₂ = -Q₁ = A. Kelly and Sackman (KS), provided a solution for the combined response 'Q' under such condition and is given by (3):

$$Q = \frac{|Q_1| + |Q_2|}{R} = \frac{2A}{R}$$
 (23)

 ${\tt R}$ is a reduction Factor to be applied to the absolute sum of the two modes and is given by:

$$R = \frac{\sqrt{1 + E^2}}{E} e^K$$
(24)

(25)

where: $K = \frac{\arctan(E)}{E}$

$$E = \frac{\Delta \omega}{2\beta}$$
(26)

 $\Delta \omega$ = the difference between the Frequencies of the two CSM

Figure (3) shows the behaviour of the reduction factor 'R' with 'E' (for a two degree-of-freedom system as studied by KS (3), $Q_1 = \frac{1}{2} \frac{I}{\sqrt{\mu}}$; $Q_2 = \frac{-1}{2\sqrt{\mu}}$; $E = \frac{\sqrt{\mu}}{2\beta}$).

For small value of E ; $R = \frac{e}{E}$ (27)

Newmark and Rosenblueth (4) presented a general method for combining a number of CSM. The method is given by the equation (using the same notations as in Ref (4)).

$$Q^{2} = \sum_{i} Q_{i}^{2} + \sum_{i \neq j} \frac{Q_{i} Q_{j}}{1 + E_{ij}^{2}}$$
(28)

For the case of $Q_1 = -Q_2 = A$;

Q =
$$\frac{\sqrt{2} E}{1+E^2}$$
 · A = $\frac{2 A}{R^2}$ (29)

where:
$$R' = \frac{\sqrt{2}\sqrt{1 + E^2}}{E}$$
 (30)

 R^{\star} is a reduction Factor to be applied to the absolute sum if the Newmark and Rosenblueth (NR) method is implemented.

For small values of E;
$$R^* = \frac{\sqrt{2}}{E}$$
 (31)

It is clear that the two reduction Factors of KS and NR are different. Fig (3) shows the behaviour of the reduction Factor 'R'' of NR with 'E'.

The most important conclusion here is that the NR method is more conservative than the KS method. For small values of E the predictability of the two methods is comparable within the factor $\frac{1}{\sqrt{2^{2}}}$

In view of this difference, it was appropriate to use the NR method which is the most conservative of the two. Another less obvious

reason is the fact that the KS method is, strictly speaking, applicable to a beat phenomenon generated by only two CSM. Extending the concept to three CSM is not a straightforward process.

AMPLIFICATION FACTORS FOR P-S-SS SYSTEM

Utilizing the mode shapes and participation factors as developed before for the tuned P-S-SS system, it is possible to develop Secondary Systems amplification factors by modal analysis. It is proposed in this modal analysis that the NR method is used and the damping is taken as the average value for the damping in the P, S and SS system. As an example, the acceleration responses of m', m, and M for a <u>unit</u> ground acceleration and a constant modal damping of 3% have been developed and are presented in Fig (4). (In this Figure the system was assumed to be in the constant acceleration branch of the ground response spectrum with an amplification factor corresponding to 3% damping of 3.5). In view of the fact that the response of m is dominated by the first and the third modes, with negligible contribution from the second, the response of m was calculated by the KS method as well, and is presented in Fig (4).

It can be observed from the Figure that the reduction in response due to mass ratios is negligible for mass ratios $(\mu+\gamma)$ lower than 0.001 (being less than 15% for m' and less than 12% for m).

For mass ratios greater than 0.001, a substantial reduction in response does occur. This reduction is more pronounced for m' (SS-system) than for m (S-system). This can be explained by the fact that the response of m is dependent on $\sqrt{(\mu+\gamma)}$ which is a somewhat slower variation. It can be observed from the Figure also, that the response of m' for the resonance case and small mass ratios can be extremely large. However, it is believed that the perfect resonance case for three systems in cascade is extremely improbable for actual nuclear power plants systems which have some nonlinearities in them. By normalizing the response of m' to the response of m, third level amplification Factors can be calculated. (These amplification Factors are analogeous to the First Level amplificiation Factors associated with the Floor Response Spectrum or the second level amplification Factors are plotted also in Figure (4).

MODAL DAMPING

When the equations of motion are uncoupled in a conventional way by using natural coordinates, the resulting damping matrix for the problem under study becomes:

[C] = modal damping matrix

$$\begin{bmatrix} \bar{C} \end{bmatrix} = \begin{bmatrix} \phi_{1}^{T} [C] \phi_{1} & \phi_{1}^{T} [C] \phi_{1} & \phi_{1}^{T} [C] \phi_{1} \\ \phi_{1}^{T} [M] \phi_{1} & \phi_{1}^{T} [M] \phi_{1} & \phi_{1}^{T} [M] \phi_{1} \end{bmatrix} \begin{bmatrix} \phi_{1} & \phi_{1}^{T} [C] \phi_{3} \\ \phi_{1}^{T} [M] \phi_{1} & \phi_{1}^{T} [M] \phi_{1} \end{bmatrix} \begin{bmatrix} \phi_{1} & \phi_{1}^{T} [M] \phi_{1} \\ \phi_{2}^{T} [C] \phi_{1} & \phi_{2}^{T} [C] \phi_{2} & \phi_{2}^{T} [C] \phi_{3} \\ \phi_{2}^{T} [M] \phi_{2} & \phi_{2}^{T} [M] \phi_{2} & \phi_{2}^{T} [M] \phi_{2} \end{bmatrix} \begin{bmatrix} \phi_{1} & \phi_{1} \\ \phi_{2}^{T} [C] \phi_{1} \\ \phi_{3}^{T} [C] \phi_{1} & \phi_{3}^{T} [C] \phi_{2} \\ \phi_{3}^{T} [M] \phi_{3} & \phi_{3}^{T} [M] \phi_{3} & \phi_{3}^{T} [M] \phi_{3} \end{bmatrix}$$
(32)

Where [C] and [M] are the conventional damping and mass matrix for the system. Modal analysis by definition involves neglecting the offdiagonal terms in the above matrix or alternatively choosing [C] matrix to make these off-diagonal terms vanish. In this case, the equations of motion become uncoupled in a conventional way, and the modal damping values β_i for the three modes are given by:

$$2\overline{\beta}_{i} \ \overline{\omega}_{i} = \frac{\phi_{i}^{T} \ [C] \ \phi_{i}}{\phi_{i}^{T} \ [M] \ \phi_{i}} \qquad (i = 1, 2, 3)$$
(33)

The above approach is consistent with the concept of modal analysis. It leads to modal damping values which are somewhat weighted average of the damping values in each component. Other weighting Functions such as the strain energy or the kinetic energy can be implemented as well. To avoid introducing artificial differences due to the implementation of an approximate modal damping scheme, an average damping value of 3% was used in the current study and was maintained throughout.

CONCLUSIONS

Based on the current work the following can be concluded:

- (a) When conventional modal analysis is applied to a coupled model consisting of the secondary systems and the primary system at resonance, the responses of the secondary systems determined by the absolute sum rule or the SRSS rule rends to ∞ when the mass ratios (µ+γ) tends to zero. Thus a conventional modal analysis utilizing a coupled model with very small mass ratios may result in both unrealistic and erroneous results for the secondary systems responses. The practice of using elaborate coupled models for seismic analysis of very light equipment and piping should not be encouraged when using conventional modal analysis.
- (b) Secondary systems responses obtained by decoupling from the primary system are always conservative. The degree of conserva-

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tism increases with the mass ratios. The degree of conservatism is more for an SS-system compared to an S-system.

- (c) The main governing parameter for the behaviour of a P-S-SS coupled system at reasonance is the sum of mass ratios (μ + γ). This is analogous to the mass ratio μ governing the behaviour of a P-S system as explained in References 1 and 2.
- (d) An upper bound response for the SS-system does exist and is equal to $\frac{1}{\sqrt{\mu+\gamma}}$ times the primary system response. An upper bound response for the S-system does exist also, and is equal to $\frac{1}{\sqrt{\mu+\gamma}}$ times the primary system response. These upper bounds represent an absolute sum for the modes and are therefore very conservative. A lower bound response for the S and SS-system is equal to the primary system response. This lower bound can be obtained by taking an agebraic sum of the modes.
- (e) The response of the primary system obtained in a coupled analysis by the SRSS rule may be deficient. To overcome this deficiency, closely spaced modes generated in this coupled analysis, should be summed by an adequate modal summation rule such as that of Newmark and Rosenblueth (4).
- (f) Reduction Factors for absolute sum response, obtained by Kelly and Sackman's method are different from those obtained by Newmark and Rosenblueth's method. The newmark and Rosenblueth's method was found to be more conservative and thus is recommended for practical applications.
- (g) Amplification Factors for a coupled P-S-SS system which accounts for mass ratio effects can be developed for a particular damping value and a particular modal combination rule. These amplification Factors such as those presented in Fig (4) show a reduction in response with the increase in mass ratios. The response reduction observed in an SS-system is more than that observed in an S-system.

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